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Comprehensive Optimization of the Tripolar Concentric Ring Electrode with Respect to the Accuracy of Laplacian Estimation Based on the Finite Dimensions Model of the Electrode †

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Abstract: Optimization performed in this study is based on the finite dimensions model of the concentric ring electrode as opposed to the negligible dimensions model widely used in the past. This makes the optimization problem comprehensive since all of the electrode parameters, including, for the first time, the radius of the central disc and individual widths of concentric rings, are optimized simultaneously. The optimization criterion used is maximizing the accuracy of the surface Laplacian estimation since the ability to estimate the Laplacian at each electrode constitutes the primary biomedical significance of concentric ring electrodes. Even though the obtained results and derived principles defining optimal electrode configurations are illustrated on tripolar (two concentric rings) electrodes, they were also confirmed for quadripolar (three rings) and pentapolar (four rings) electrodes and are likely to continue to hold for any higher number of concentric rings. For tripolar concentric ring electrodes, the optimal configuration was compared to previously proposed, linearly increasing inter-ring distances and constant inter-ring distances in configurations of the same size and based on the same finite dimensions model of the electrode. The obtained results suggest that previously proposed configurations correspond to almost two-fold and more than three-fold increases in Laplacian estimation error, respectively, compared to the optimal configuration proposed in this study.

Keywords: electrophysiology; measurement; wearable sensors; noninvasive; concentric ring electrodes; Laplacian; estimation; optimization

1. Introduction

Concentric ring electrodes (CREs) are noninvasive electrodes for electrophysiological measurement, with their primary biomedical significance tied to their ability to accurately estimate the Laplacian (second spatial derivative of the surface potential) at each electrode. Properties shared by the majority of currently used CREs are as follows: relatively small radius of the central disc (compared to the radius of the electrode) and/or equal and small widths of concentric rings (compared to the radius of the electrode) [1–8]. These properties stem, at least partially, from the use of the negligible dimensions model (NDM) of a CRE—a Cartesian grid where the central disc is represented by a single point (of negligible diameter) in the middle of the grid and rings are represented by concentric circles (of negligible width) around it. For example, since NDM was used to calculate Laplacian estimates for tripolar CRE (TCRE) in [9], it also influenced the design of respective physical electrodes. Previous results on improving the Laplacian estimation accuracy via CRE optimization were also based on NDM [10–12].

The first proof of concept of the finite dimensions model (FDM) of the CRE with nonnegligible widths of concentric rings and the radius of the central disc was introduced in [13] before being developed into a comparison framework validated on human electrocardiogram data in [5]. This framework, allowing direct comparison between two CRE configurations with the same number of rings and the same size but with different radii of the central disc, widths of concentric rings, and inter-ring distances was used in this study to define and solve a comprehensive CRE optimization problem, maximizing the Laplacian estimation accuracy via said CRE. Unlike the NDM-based optimization problem that was solved in [12], this study includes and optimizes all the CRE parameters simultaneously. Absolute values of truncation term coefficients of the lowest remaining order were compared, since, in [11,12], ratios of these coefficients have been shown, using finite element method modeling, to be predictors of the Laplacian estimation error. Specifically, ratios of relative and maximum errors of Laplacian estimation calculated using finite element method modeling and analytic ratios of truncation term coefficients differed by less than 5% for combinations of linearly increasing inter-ring distances (LIIRD), constant inter-ring distances (CIRD), and linearly decreasing inter-ring distances TCREs, and quadripolar CREs [11], as well as for their quadratically increasing inter-ring distance counterparts [12]. Moreover, in [5], consistency between NDM and FDM in terms of values of truncation term coefficient ratios have been demonstrated for CIRD and LIIRD TCRE configurations. This is to be expected since NDM and FDM are also consistent in terms of the highest order of the truncation term that can be canceled out during derivation of the Laplacian estimate that has been shown to be equal to twice the number of concentric rings in the electrode in [10] (for NDM) and [13] (for FDM), respectively.

As a result of this study, general principles defining optimal CRE configurations maximizing the accuracy of Laplacian estimation are defined and illustrated for the case of TCREs. Moreover, optimal TCRE configuration is directly compared to LIIRD and CIRD configurations from [5]. CIRD configuration from [5] corresponds to a more than three-fold increase in Laplacian estimation error while LIIRD configuration from [5] corresponds to an almost two-fold increase in Laplacian estimation error compared to the optimal TCRE configuration proposed in this study.

2. Materials and Methods

2.1. Preliminaries

Figure 1 represents the FDM diagrams of three TCRE configurations including CIRD (Figure 1A) and LIIRD (Figure 1B) ones that were used to illustrate the comparison framework in [5]. All three configurations in Figure 1 have the same radius subdivided into 9 equal intervals. CIRD and LIIRD configurations have the radius of the central disc and widths of both rings equal to 1/9 of the electrode radius. For CIRD configuration, both distance between the central disc and the middle ring and distance between the middle ring and the outer ring are equal to 3/9 (= 1/3) of the electrode radius. For LIIRD configuration, the distance between the central disc and the middle ring (2/9 of the electrode radius) is one half of the distance between the middle ring and the outer ring (4/9 of the electrode radius). Average potential on each concentric circle with the radius ranging from 1 to 9 is calculated using Huiskamp's Laplacian potential derivation based on the Taylor series expansion from [14]. Main steps of the comparison framework for TCRE configuration are listed below (see [5] for more detail) with similar steps used for quadripolar, pentapolar, etc., configurations:

1. Calculating the potentials on all three recording surfaces (central disc and two concentric rings) of the TCRE. For example, the potential on the central disc in all three TCRE configurations in Figure 1 is equal to the average of the potential at the center of the central disc and the potential on the concentric circle with radius equal to 1/9 of the electrode radius.
2. Canceling out the potential at the center of the central disc by taking bipolar differences between potentials on the middle ring and on the central disc and between potentials on the outer ring and on the central disc, respectively.
3. Combining the two bipolar differences linearly to cancel out the 4th (twice the number of concentric rings) order truncation term, solve for the surface Laplacian estimate, and to calculate

the absolute value of the 6th order truncation term coefficient (lowest remaining truncation term order for TCRE). Lowest remaining truncation term order is used since "higher-order terms usually contribute negligibly to the final sum and can be justifiably discarded" from the Taylor series [15].

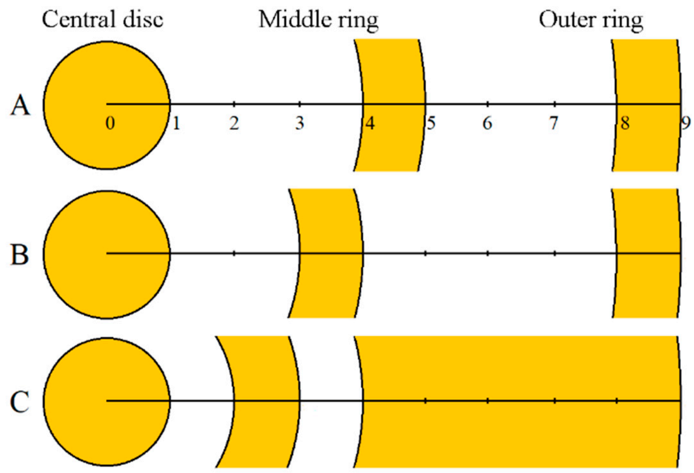

Figure 1. Finite dimensions models of three tripolar concentric ring electrode configurations including: (**A**) constant inter-ring distance configuration, (**B**) linearly increasing inter-ring distance configuration, and (**C**) optimal configuration with respect to the accuracy of Laplacian estimation.

2.2. Optimization Problem

Comparison framework from [5] has been developed into a comprehensive optimization problem directly comparing not pairs but all the possible CRE configurations of the same size and with the same number of rings simultaneously. Absolute values of truncation term coefficients for the lowest remaining truncation term order were calculated for CRE configurations of given electrode size and with given number of rings including all the possible combinations of values for the radius of the central disc, widths of concentric rings, and inter-ring distances. The lowest absolute value of the truncation term coefficient corresponds to the highest accuracy of Laplacian estimation and vice versa.

3. Results

3.1. General Principles Defining Optimal CRE Configurations

Before the general principles that define optimal CRE configurations maximizing the accuracy of Laplacian estimation are introduced, the results of optimization for TCRE with the outer radius of the outer ring (the electrode radius) equal to 6 are presented in Table 1. These results will be used to illustrate each of the aforementioned principles.

Table 1. All possible tripolar concentric ring electrode configurations for the outer radius of the outer ring equal to 6.

TCRE Number	Central Disc Radius	Middle Ring Radii		Outer Ring Radii		Absolute Value of the 6th Order Truncation Term Coefficient	Increase with Respect to the Optimal (%)
		Inner	Outer	Inner	Outer		
1	1	2	3	4	6	0.685	0
2	1	2	3	5	6	0.717	4.65
3	1	2	4	5	6	1.096	59.99
4	1	3	4	5	6	1.250	82.53
5	2	3	4	5	6	1.369	99.93

Table 1 contains all five possible TCRE configurations sorted in accordance with the respective absolute values of the sixth order truncation term coefficients whose ratios have been shown to be predictors of the Laplacian estimation error in [11,12] (hence, the two terms are used interchangeably below). Percentage of increase in the absolute value of the sixth order truncation term coefficient with respect to the optimal configuration (TCRE configuration number 1) is also provided in the rightmost column of Table 1. It can be seen from Table 1 that even for such a small electrode radius of 6 (reducing it to 5 results in a single possible TCRE configuration), the difference between the Laplacian estimation errors for the optimal and the worst case scenario TCRE configurations (TCRE configuration number 5) approach 100%.

General principles defining optimal CRE configurations (independent of the number of concentric rings) in terms of accuracy of the surface Laplacian estimate are as follows:

1. In the optimal configuration, central disc and concentric rings are kept at minimum distances with minimum radius/widths except for the width of the outer ring.
   Example: TCRE configuration number 1 in Table 1.
2. Larger width of the outer ring is advantageous to smaller width in electrode configurations that are otherwise identical.
   Example: TCRE configuration number 1 versus number 2 in Table 1.
3. Increasing the width of a concentric ring closer to the outer edge of the electrode is advantageous to increasing the width of a concentric ring closer to the central disc.
   Example: TCRE configuration number 2 versus numbers 1 and 3 in Table 1.
4. Increasing the width of any concentric ring is advantageous to increasing the radius of the central disc.
   Example: TCRE configuration number 1 versus numbers 3 and 5 in Table 1.
5. Increasing the distance between recording surfaces closer to the outer edge is advantageous to increasing the distance between recording surfaces closer to the central disc.
   Example: TCRE configuration number 2 versus number 4 in Table 1.

3.2. Comparison of the Optimal TCRE Configuration with Previous Results

Results of FDM optimization for TCREs are compared with our previously obtained results in Table 2 that correspond to the outer radius of the outer ring (the electrode radius) equal to 9.

Table 2. Selected tripolar concentric ring electrode configurations for the outer radius of the outer ring equal to 9.

TCRE Number	Central Disc Radius	Middle Ring Radii		Outer Ring Radii		Absolute Value of the 6th Order Truncation Term Coefficient	Increase with Respect to the Optimal (%)
		Inner	Outer	Inner	Outer		
1	1	2	3	4	9	1.447	0
2	1	2	3	5	9	1.458	0.78
3	1	2	3	6	9	1.489	2.94
4	1	2	3	7	9	1.550	7.19
5	1	2	3	8	9	1.650	14.07
…	…	…	…	…	…	…	…
15	1	3	4	8	9	2.883	99.33
…	…	…	…	…	…	…	…
30	1	4	5	8	9	4.528	213.01
…	…	…	…	…	…	…	…
66	4	5	7	8	9	9.189	535.22
67	2	6	7	8	9	9.407	550.35
68	3	6	7	8	9	9.901	584.45
69	4	6	7	8	9	10.436	621.46
70	5	6	7	8	9	10.879	652.05

Out of the total of 70 possible TCRE configurations with radius equal to 9, Table 2 presents the top five, the bottom five, and two TCRE configurations assessed in [5]: CIRD (TCRE configuration number 30; Figure 1A) and LIIRD (TCRE configuration number 15; Figure 1B). While the results in Table 2 follow the same general principles defining optimal CRE configurations as the results in Table 1, the difference between the Laplacian estimation errors for the optimal and the worst case scenario TCRE configurations increased to over 650% in Table 2 compared to under 100% in Table 1. This increase of more than 6.5 times is due to the mere 1.5 times increase in the electrode radius (from 6 in Table 1 to 9 in Table 2). More importantly, in direct comparison, the optimal TCRE configuration (TCRE configuration number 1 in Table 2; Figure 1C) outperforms two TCRE configurations from [5] by 99.33% and 213.01%, respectively, in terms of the Laplacian estimation error.

4. Discussion

In this study optimizing the FDM-based TCRE configurations with respect to the accuracy of Laplacian estimation (Tables 1 and 2), the distinctive feature of the obtained results is that in optimal TCRE configurations, the recording surfaces account for the vast majority of the electrode surface area, minimizing the distances between the recording surfaces (e.g., optimal TCRE configuration in Figure 1C). This is markedly different from the currently used CREs, where the majority of the electrode surface area corresponds to the distances between the recording surfaces (for example, see CREs from [5,6]). Compared to the optimal TCRE configuration (Figure 1C), the LIIRD configuration of the same size (Figure 1B) increases the Laplacian estimation error almost two-fold, while the CIRD configuration (Figure 1A) corresponds to a more than three-fold increase. These results have the potential to inform the design of future CREs and could not have been obtained with simplistic NDM.

At the same time, some aspects of FDM-based optimal configurations are consistent with the previous results obtained using NDM. For example, locating the middle ring closer to the central disc than to the outer ring is consistent with analytical and finite element method modeling results from [11,12]. Another example is an increase in Laplacian estimation error associated with an increase in the size of the electrode observed via modeling in [10–12]. Similar results can be seen in Tables 1 and 2, corresponding to the TCRE radius increasing from 6 to 9, with the optimal absolute value of the sixth order truncation term coefficient increasing from 0.685 to 1.447, respectively.

The only optimization criterion used in this study was maximizing the accuracy of surface Laplacian estimation via the CRE. Other optimization criteria may result in different optimal electrode configurations so adding additional optimization criteria to the optimization problem solved in this study is one of the directions of future work. More importantly, for optimal CRE configurations, the question of how small the distances between the recording surfaces can be before shorting due to salt bridges negatively affects the accuracy of Laplacian estimation becomes more critical than before, since the first principle defining optimal configurations is to ensure that these distances are minimal. Modeling or prototyping of optimal CRE configurations is needed to answer this question.

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