Proof of concept Laplacian estimate derived for noninvasive tripolar concentric ring electrode with incorporated radius of the central disc and the widths of the concentric rings*

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Abstract— **Tripolar concentric ring electrodes are showing great promise in a range of applications including brain-computer interface and seizure onset detection due to their superiority to conventional disc electrodes, in particular, in accuracy of surface Laplacian estimation. Recently, we proposed a general approach to estimation of the Laplacian for an (n + 1)-polar electrode with n rings using the (4n + 1)-point method for $n \geq 2$ that allows cancellation of all the truncation terms up to the order of 2n. This approach has been used to introduce novel multipolar and variable inter-ring distances concentric ring electrode configurations verified using finite element method. The obtained results suggest their potential to improve Laplacian estimation compared to currently used constant inter-ring distances tripolar concentric ring electrodes. One of the main limitations of the proposed (4n + 1)-point method is that the radius of the central disc and the widths of the concentric rings are not included and therefore cannot be optimized. This study incorporates these two parameters by representing the central disc and both concentric rings as clusters of points with specific radius and widths respectively as opposed to the currently used single point and concentric circles. A proof of concept Laplacian estimate is derived for a tripolar concentric ring electrode with non-negligible radius of the central disc and non-negligible widths of the concentric rings clearly demonstrating how both of these parameters can be incorporated into the (4n + 1)-point method.**

I. Introduction

Electroencephalography (EEG) is an essential tool for brain and behavioral research as well as one of the mainstays of hospital diagnostic procedures and pre-surgical planning. Despite scalp EEG's many advantages end users struggle with its poor spatial resolution, selectivity and low signal-to-noise ratio that are critically limiting the research discovery and diagnosis [1]–[3]. In particular, EEG's poor spatial resolution is primarily due to (1) the blurring effects of the volume conductor with disc electrodes; and (2) EEG signals having reference electrode problems as idealized references are not available with EEG and interference on the reference electrode contaminates all other electrode signals [2]. The application of the surface Laplacian (the second spatial derivative of the potentials on the scalp surface) to EEG has been shown to alleviate the blurring effects enhancing the spatial resolution and selectivity, and reduce the reference problem [4]–[6].

Noninvasive concentric ring electrodes (CREs) can resolve the reference electrode problems since they act like closely spaced bipolar recordings [2]. Moreover, CREs are symmetrical alleviating electrode orientation problems [7]. They also act as spatial filters reducing the low spatial frequencies and increasing the spatial selectivity [7], [8]. Most importantly, tripolar CREs (TCREs; Fig. 1B) have been shown to estimate the surface Laplacian directly through the nine-point method, an extension of the five-point method (FPM) used for bipolar CREs, and significantly better than other electrode systems including bipolar and quasi-bipolar CRE configurations [9], [10]. Compared to EEG with conventional disc electrodes (Fig. 1A) Laplacian EEG via TCREs (tEEG) have been shown to have significantly better spatial selectivity (approximately 2.5 times higher), signal-to-noise ratio (approximately 3.7 times higher), and mutual information (approximately 12 times lower) [11]. Because of such unique capabilities TCREs have found numerous applications in a wide range of areas including brain–computer interface [12], [13], seizure onset detection [14], [15], detection of high-frequency oscillations and seizure onset zones [16], etc.

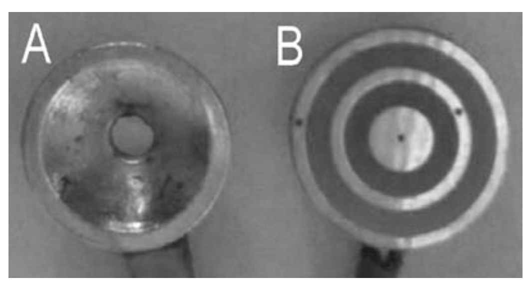

Figure 1. Conventional disc electrode (A) and tripolar concentric ring electrode (B).

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In [17] we have shown that accuracy of Laplacian estimation can be improved with multipolar CREs. General approach to estimation of the Laplacian for an $(n + 1)$-polar electrode with n rings using the $(4n + 1)$-point method for $n \geq 2$ has been proposed. This approach allows cancellation of all the Taylor series truncation terms up to the order of $2n$ which has been shown to be the highest order achievable for a CRE with n rings [17]. Proposed approach was validated using finite element method (FEM) modeling. Multipolar CRE configurations with n ranging from 1 ring (bipolar configuration) to 6 rings (septapolar configuration) were compared and obtained results suggested statistical significance of the increase in Laplacian accuracy caused by increase in the number of rings n [17].

Most recently, in [18] the next fundamental step toward further improving the Laplacian estimation accuracy was taken by proposing novel variable inter-ring distances CREs. Laplacian estimates for linearly increasing and linearly decreasing inter-ring distances TCRE ($n = 2$) and quadripolar CRE ($n = 3$) configurations were derived using a modified $(4n + 1)$-point method from [17] and directly compared to their constant inter-ring distances counterparts using Laplacian estimation errors obtained via FEM modeling. The obtained results suggested that increasing inter-ring distances CRE configurations may offer more accurate Laplacian estimates compared to respective constant inter-ring distances CRE configurations.

One of the main limitations of the $(4n + 1)$-point method is that at this point of time the widths of the concentric rings and the radius of the central disc are not taken into account and therefore cannot be optimized [17], [18]. Moreover, assuming these parameters to be negligible is inconsistent with the physical design of currently used TCREs (Fig. 1B). To pursue our ultimate goal of optimizing CRE designs for specific applications, in this study, these parameters are included into the modified $(4n + 1)$-point method along with the currently included number of rings, size of the electrode, and inter-ring distances. A proof of concept surface Laplacian estimate is derived for a specific TCRE configuration.

II. Methods

A. Preliminaries and Notations

In [17] the general $(4n + 1)$-point method for constant inter-ring distances $(n + 1)$-polar CRE was derived using a regular plane square grid with all inter-point distances equal to r as shown in Fig. 2. As one of the steps in deriving the method, FPM (a bipolar CRE configuration Laplacian estimate) was applied to the points with potentials v_0, $v_{r,1}$, $v_{r,2}$, $v_{r,3}$ and $v_{r,4}$ (first concentric ring of radius r) following Huiskamp's calculation of the Laplacian potential using Taylor series expansion [19]:

$$\sum_{i=1}^{4} v_{r,i} = 4v_0 + \frac{2r^2}{2!}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{2r^4}{4!}\left(\frac{\partial^4 v}{\partial x^4} + \frac{\partial^4 v}{\partial y^4}\right) + \ldots \quad (1)$$

where $\frac{2r^4}{4!}\left(\frac{\partial^4 v}{\partial x^4} + \frac{\partial^4 v}{\partial y^4}\right) + \frac{2r^6}{6!}\left(\frac{\partial^6 v}{\partial x^6} + \frac{\partial^6 v}{\partial y^6}\right) \ldots$

is the truncation error. Similar FPMs were applied to the points with potentials v_0, $v_{2r,1}$, $v_{2r,2}$, $v_{2r,3}$ and $v_{2r,4}$ (second concentric ring of radius $2r$), etc. For the outer ring of the $(n + 1)$-polar CRE with radius nr the FPM was applied to the points with potentials v_0, $v_{nr,1}$, $v_{nr,2}$, $v_{nr,3}$ and $v_{nr,4}$ (Fig. 2). All the FPMs including the one in (1) can be further generalized by taking the integral along the circle of the respective radius (equal to r in case of (1)) around the point with potential v_0 of the Taylor series expansion but for the purposes of this study using averages of potentials from points on the circle is sufficient.

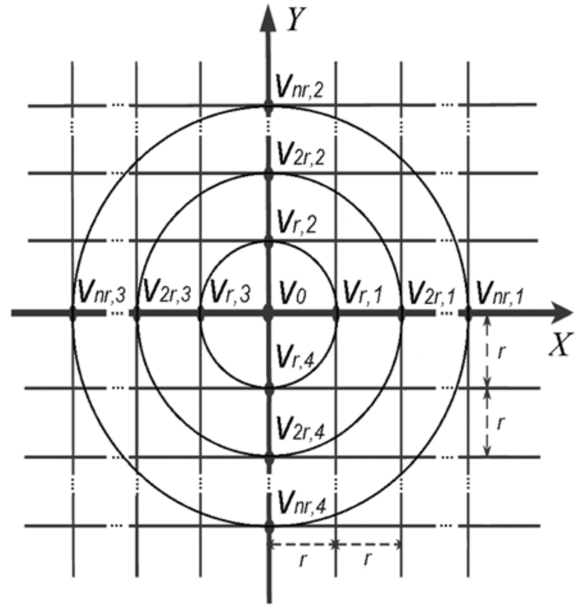

Figure 2. Regular plane square grid with inter-point distances equal to r.

B. Proof of concept TCRE configuration

In Fig. 2, the central disc of the CRE is represented as a point with negligible radius and the concentric rings are represented as circles with negligible widths. A proof of concept TCRE configuration with non-negligible central disc radius and non-negligible widths of concentric rings is presented in Fig. 3. In this proof of concept configuration, the central disc has a radius of $2r$ and both concentric rings have widths of $2r$.

In Table 1, the $(4n + 1)$-point method notations from Fig. 2 are used to describe the average potentials on concentric circles of radii ranging from r to $8r$ for the proof of concept TCRE in Fig. 3.

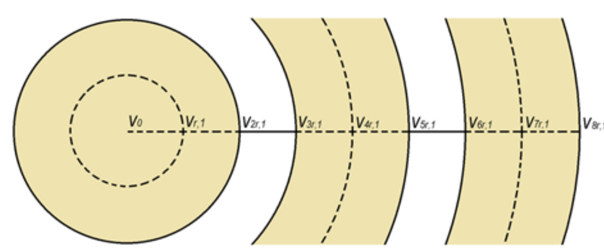

Figure 3. Proof of concept TCRE with non-negligible central disc radius and concentric ring width.

TABLE I. AVERAGE POTENTIALS ON CONCENTRIC CIRCLES

Concentric Circle Radius	Average Potential on Concentric Circle
r	$v_r = \frac{1}{4}\sum_{i=1}^{4} v_{r,i}$
$2r$	$v_{2r} = \frac{1}{4}\sum_{i=1}^{4} v_{2r,i}$
$3r$	$v_{3r} = \frac{1}{4}\sum_{i=1}^{4} v_{3r,i}$
$4r$	$v_{4r} = \frac{1}{4}\sum_{i=1}^{4} v_{4r,i}$
$5r$	$v_{5r} = \frac{1}{4}\sum_{i=1}^{4} v_{5r,i}$
$6r$	$v_{6r} = \frac{1}{4}\sum_{i=1}^{4} v_{6r,i}$
$7r$	$v_{7r} = \frac{1}{4}\sum_{i=1}^{4} v_{7r,i}$
$8r$	$v_{8r} = \frac{1}{4}\sum_{i=1}^{4} v_{8r,i}$

In Table 2 the same average concentric circle potentials v_{kr} for the concentric rings with radius kr for k ranging from 1 to 8 (Table 1) are solved for using corresponding FPM Taylor series expansion equations similar to (1).

TABLE II. TABLE TYPE STYLES

Concentric Circle Radius	Taylor Series for Concentric Circle
r	$v_r \cong v_0 + \frac{2\cdot 1^2}{4\cdot 2!} r^2 \Delta v_0 + \frac{2\cdot 1^4}{4\cdot 4!} r^4 T_4$
$2r$	$v_{2r} \cong v_0 + \frac{2\cdot 2^2}{4\cdot 2!} r^2 \Delta v_0 + \frac{2\cdot 2^4}{4\cdot 4!} r^4 T_4$
$3r$	$v_{3r} \cong v_0 + \frac{2\cdot 3^2}{4\cdot 2!} r^2 \Delta v_0 + \frac{2\cdot 3^4}{4\cdot 4!} r^4 T_4$
$4r$	$v_{4r} \cong v_0 + \frac{2\cdot 4^2}{4\cdot 2!} r^2 \Delta v_0 + \frac{2\cdot 4^4}{4\cdot 4!} r^4 T_4$
$5r$	$v_{5r} \cong v_0 + \frac{2\cdot 5^2}{4\cdot 2!} r^2 \Delta v_0 + \frac{2\cdot 5^4}{4\cdot 4!} r^4 T_4$
$6r$	$v_{6r} \cong v_0 + \frac{2\cdot 6^2}{4\cdot 2!} r^2 \Delta v_0 + \frac{2\cdot 6^4}{4\cdot 4!} r^4 T_4$
$7r$	$v_{7r} \cong v_0 + \frac{2\cdot 7^2}{4\cdot 2!} r^2 \Delta v_0 + \frac{2\cdot 7^4}{4\cdot 4!} r^4 T_4$
$8r$	$v_{8r} \cong v_0 + \frac{2\cdot 8^2}{4\cdot 2!} r^2 \Delta v_0 + \frac{2\cdot 8^4}{4\cdot 4!} r^4 T_4$

where $\Delta v_0 = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$ is the surface Laplacian potential at the point with potential v_0 and $T_4 = \frac{\partial^4 v}{\partial x^4} + \frac{\partial^4 v}{\partial y^4}$ is the fourth order truncation term.

III. RESULTS

In order to derive the surface Laplacian estimate for the proof of concept TCRE configuration from Fig. 3 we first calculate the potentials on three recording surfaces of the TCRE: the central disc, the middle concentric ring, and the outer concentric ring.

To calculate the potential v_{CD} on the central disc with radius equal to $2r$ (Fig. 3) we average the potentials at the center point of the square grid with potential v_0, on the concentric circle of radius r, and on the concentric circle of radius $2r$:

$$v_{CD} = \frac{v_0 + v_r + v_{2r}}{3} \cong v_0 + \frac{5}{12} r^2 \Delta v_0 + \frac{17}{144} r^4 T_4 \tag{2}$$

To calculate the potential v_{MR} on the middle concentric ring of the TCRE (Fig. 3) we average the potentials on concentric circles with radii equal $3r$, $4r$, and $5r$ as follows:

$$v_{MR} = \frac{v_{3r} + v_{4r} + v_{5r}}{3} \cong v_0 + \frac{25}{6} r^2 \Delta v_0 + \frac{481}{72} r^4 T_4 \tag{3}$$

To calculate the potential v_{OR} on the outer concentric ring of the TCRE (Fig. 3) we average the potentials on concentric circles with radii equal $6r$, $7r$, and $8r$ as follows:

$$v_{OR} = \frac{v_{6r} + v_{7r} + v_{8r}}{3} \cong v_0 + \frac{149}{12} r^2 \Delta v_0 + \frac{7793}{144} r^4 T_4 \tag{4}$$

Next, we subtract the potential on the central disc v_{CD} (2) from the potential on the inner concentric ring v_{MR} (3) to cancel out v_0:

$$v_{MR} - v_{CD} \cong \frac{15}{4} r^2 \Delta v_0 + \frac{105}{16} r^4 T_4 \tag{5}$$

In a similar way, we subtract the potential on the central disc v_{CD} (2) from the potential on the outer concentric ring v_{OR} (4) to cancel out v_0:

$$v_{OR} - v_{CD} \cong 12 r^2 \Delta v_0 + 54 r^4 T_4 \tag{6}$$

Finally, we combine differences (5) and (6) into a linear combination with coefficients 24/55 and -7/132 respectively to cancel out the fourth order truncation term T_4 and provide the estimate for Laplacian Δv_0:

$$\begin{aligned} \Delta v_0 &\cong \frac{1}{r^2}\left[\frac{24}{55}\left(v_{MR} - v_{CD}\right) - \frac{7}{132}\left(v_{OR} - v_{CD}\right)\right] \\ &= \frac{1}{7260 r^2}\left[3168\left(v_{MR} - v_{CD}\right) - 385\left(v_{OR} - v_{CD}\right)\right] \end{aligned} \tag{7}$$

IV. DISCUSSION

This work is a part of our continued effort to improve the electrode design for noninvasive electrophysiology via CREs with our ultimate goal being determining suboptimal CRE designs for specific applications. The surface Laplacian estimate in (7) is, to the best of the authors' knowledge, the first one derived taking into account the widths of the concentric rings and the radius of the central disc. Similar steps can be used to estimate Laplacian for any other multipolar

CRE configuration where these two parameters are non-negligible.

The highest order of truncation term that was cancelled in Laplacian estimate (7) was the fourth order which is consistent with the original ($4n + 1$)-point method that allows cancellation of all the Taylor series truncation terms up to the order of $2n$ ($2n = 4$ for $n = 2$ in case of TCRE) [17].

Alternative approach to incorporating these two parameters could potentially be based on using the areas of the central disc and the concentric rings as an area of a circle and a difference between the areas of two circles respectively. However, this alternative approach is likely to require going beyond Taylor series expansion to cancel out terms such as v_0 in Laplacian estimate derivation.

Further investigation is needed to confirm the obtained results using FEM model modified accordingly to the proposed modification of the ($4n + 1$)-point method to incorporate two new parameters.

V. Conclusion

A proof of concept Laplacian estimate was derived for a tripolar concentric ring electrode with non-negligible radius of the central disc and non-negligible widths of the concentric establishing steps needed to incorporate these two parameters into the ($4n + 1$)-point method of surface Laplacian estimation along with the currently included number of rings, size of the electrode, and inter-ring distances.

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