

Finite element method modeling to assess Laplacian estimates via novel variable inter-ring distances concentric ring electrodes

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Abstract— Noninvasive concentric ring electrodes are a promising alternative to conventional disc electrodes. Currently, superiority of tripolar concentric ring electrodes over disc electrodes, in particular, in accuracy of Laplacian estimation has been demonstrated in a range of applications. In our recent work we have shown that accuracy of Laplacian estimation can be improved with multipolar concentric ring electrodes using a general approach to estimation of the Laplacian for an $(n + 1)$ -polar electrode with n rings using the $(4n + 1)$ -point method for $n \geq 2$. This paper takes the next step toward further improving the Laplacian estimate by proposing novel variable inter-ring distances concentric ring electrodes. Derived using a modified $(4n + 1)$ -point method, linearly increasing and decreasing inter-ring distances tripolar ($n = 2$) and quadripolar ($n = 3$) electrode configurations are compared to their constant inter-ring distances counterparts using finite element method modeling. Obtained results suggest that increasing inter-ring distances electrode configurations may decrease the estimation error resulting in more accurate Laplacian estimates compared to respective constant inter-ring distances configurations. For currently used tripolar electrode configuration the estimation error may be decreased more than two-fold while for the quadripolar configuration more than six-fold decrease is expected.

I. INTRODUCTION

Electroencephalography (EEG) is an essential tool for brain and behavioral research as well as one of the mainstays of hospital diagnostic procedures and pre-surgical planning. Despite scalp EEG's many advantages end users struggle with its poor spatial resolution, selectivity and low signal-to-noise ratio that are critically limiting the research discovery and diagnosis [1]–[3]. In particular, EEG's poor spatial resolution is primarily due to (1) the blurring effects of the volume conductor with disc electrodes; and (2) EEG signals having reference electrode problems as idealized references are not available with EEG and interference on the reference electrode contaminates all other electrode signals [2]. The application of the surface Laplacian (the second spatial derivative of the potentials on the scalp surface) to EEG has been shown to alleviate the blurring effects enhancing the

spatial resolution and selectivity, and reduce the reference problem [4]–[6].

Noninvasive concentric ring electrodes (CREs) can resolve the reference electrode problems since they act like closely spaced bipolar recordings [2]. Moreover, CREs are symmetrical alleviating electrode orientation problems [7]. They also act as spatial filters reducing the low spatial frequencies and increasing the spatial selectivity [7], [8]. Most importantly, tripolar CREs (TCREs; Fig. 1B) have been shown to estimate the surface Laplacian directly through the nine-point method, an extension of the five-point method (FPM) used for bipolar CREs, and significantly better than other electrode systems including bipolar and quasi-bipolar CRE configurations [9], [10]. Compared to EEG with conventional disc electrodes (Fig. 1A) Laplacian EEG via TCREs have been shown to have significantly better spatial selectivity (approximately 2.5 times higher), signal-to-noise ratio (approximately 3.7 times higher), and mutual information (approximately 12 times lower) [11]. Because of such unique capabilities TCREs have found numerous applications in a wide range of areas including brain-computer interface [12], [13], seizure onset detection [14], [15], detection of high-frequency oscillations and seizure onset zones [16], etc. These EEG applications of TCREs suggest the potential of CRE technology as well as the need for further improvement of CRE design.

In [17] we have shown that accuracy of Laplacian estimation can be improved with multipolar CREs. General approach to estimation of the Laplacian for an $(n + 1)$ -polar electrode with n rings using the $(4n + 1)$ -point method for $n \geq 2$ has been proposed. This approach allows cancellation of all the Taylor series truncation terms up to the order of $2n$ which has been shown to be the highest order achievable for a CRE with n rings [17]. Proposed approach was validated using finite element method (FEM) modeling. Multipolar concentric ring electrode configurations with n ranging from 1 ring (bipolar electrode configuration) to 6 rings (septapolar electrode configuration) were compared and obtained results suggested statistical significance of the increase in Laplacian accuracy caused by increase in the number of rings n [17].

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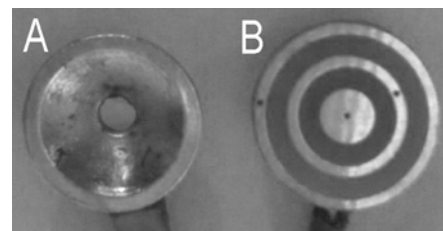


Figure 1. Conventional disc electrode (A) and tripolar concentric ring electrode (B).

To the best of the authors' knowledge, all the previous research on CREs was based on the assumption of constant inter-ring distances (distances between consecutive rings). This means that distances between the rings were not considered as a means of improving the accuracy of Laplacian estimation. This paper takes the next fundamental step toward further improving the Laplacian estimation accuracy by proposing novel variable inter-ring distances concentric ring electrodes. Laplacian estimates for linearly increasing and linearly decreasing inter-ring distances TCRE ($n = 2$) and quadripolar CRE (QCRE; $n = 3$) configurations are derived using a modified $(4n + 1)$ -point method from [17] and directly compared to their constant inter-ring distances counterparts using Laplacian estimation errors obtained via FEM modeling. Obtained results suggest that increasing inter-ring distances CRE configurations may offer more accurate Laplacian estimates compared to respective constant inter-ring distances CRE configurations.

II. MATERIALS AND METHODS

A. Notations and Preliminaries

In [17] general $(4n + 1)$ -point method for constant inter-ring distances $(n + 1)$ -polar CRE with n rings was proposed. It was derived using a regular plane square grid with all inter-point distances equal to r presented in Fig. 2.

It has been shown that for a case of multipolar CRE with n rings ($n \geq 2$) we obtain a set of n FPM equations for Laplacian potential Δv_0 , one for each ring with radii ranging from r ($v_0, v_{r,1}, v_{r,2}, v_{r,3},$ and $v_{r,4}$ on Fig. 2) to nr ($v_0, v_{nr,1}, v_{nr,2}, v_{nr,3},$ and $v_{nr,4}$ on Fig. 2) around the point with potential v_0 [17]:

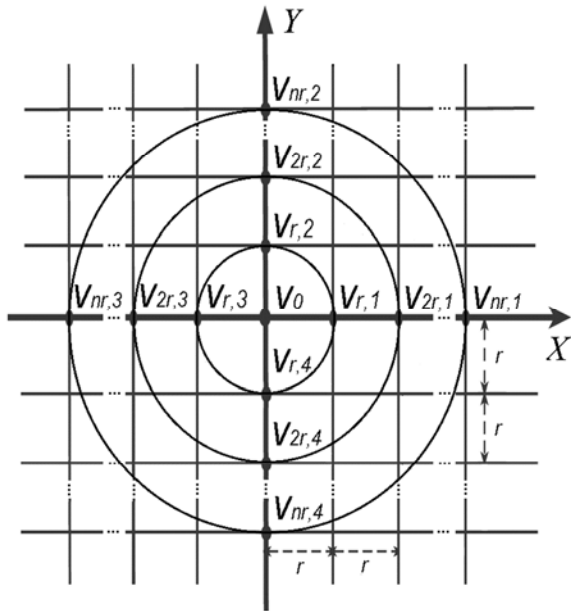


Figure 2. Regular plane square grid with inter-point distances equal to r .

$$\begin{aligned} \frac{1}{2\pi} \int_0^{2\pi} v(nr, \theta) d\theta - v_0 &= \frac{(nr)^2}{4} \Delta v_0 \\ &+ \frac{(nr)^4}{4!} \int_0^{2\pi} \sum_{j=0}^4 \sin^{4-j}(\theta) \cos^j(\theta) d\theta \left(\frac{\partial^4 v}{\partial x^{4-j} \partial y^j} \right) \\ &+ \frac{(nr)^6}{6!} \int_0^{2\pi} \sum_{j=0}^6 \sin^{6-j}(\theta) \cos^j(\theta) d\theta \left(\frac{\partial^6 v}{\partial x^{6-j} \partial y^j} \right) + \dots \end{aligned} \quad (1)$$

To estimate the Laplacian for this general case the n equations are combined in a way that cancels all the truncation terms up to the highest order that can be achieved for n rings (equal to $2n$ as shown in [17]) increasing the accuracy of the Laplacian estimate. In order to find such a combination we arrange the coefficients l^k of the truncation terms $\frac{(lr)^k}{k!} \int_0^{2\pi} \sum_{j=0}^k \sin^{k-j}(\theta) \cos^j(\theta) d\theta \left(\frac{\partial^k v}{\partial x^{k-j} \partial y^j} \right)$ for order k ranging in increments of 2 from 4 to $2n$ and ring radius multiplier l ranging from 1 to n into an $(n - 1) \times n$ matrix A that is a function only of the number of the rings n :

$$A = \begin{pmatrix} 1 & 2^4 & \dots & n^4 \\ 1 & 2^6 & \dots & n^6 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2^{2n} & \dots & n^{2n} \end{pmatrix} \quad (2)$$

A matrix equation of the form $A\bar{x} = \bar{0}$ is equivalent to a homogeneous system of linear equations where $\bar{0}$ is the $(n - 1)$ -dimensional zero vector and \bar{x} is the n -dimensional vector that allows the cancellation of all the truncation terms up to the order of $2n$ by setting the linear combination of n coefficients l^k corresponding to all ring radii for each order k equal to 0 [17].

B. Variable (Linearly Increasing and Linearly Decreasing) Inter-ring Distances CREs

We consider the case of CRE configurations with variable inter-ring distances that increase or decrease linearly the further the concentric ring lies from the central disc. To modify the $(4n + 1)$ -point method from [17] to the case of linearly increasing inter-ring distances, the distance between the central point with potential v_0 and four points on the smallest concentric ring is set equal to r . The distance between the first and the second smallest concentric ring is set equal to $2r$, etc. In this case the sum of all the inter-ring distances to the largest, n -th, outer ring can be obtained using the formula for the n -th term of the triangular number sequence that describes the sum of all points in a triangular grid where the first row contains a single point and each subsequent row contains one more point than the previous one to be equal to $n(n + 1)/2$. Therefore, modified matrix A of truncation term coefficients l^k from (2) for linearly increasing inter-ring distances CRE is equal to:

$$A' = \begin{pmatrix} 1 & 3^4 & \dots & \left(\frac{n(n+1)}{2}\right)^4 \\ 1 & 3^6 & \dots & \left(\frac{n(n+1)}{2}\right)^6 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 3^{2n} & \dots & \left(\frac{n(n+1)}{2}\right)^{2n} \end{pmatrix} \quad (3)$$

In the opposite case of CRE configuration with inter-ring distances decreasing linearly the further the concentric ring lies from the central disc the distance between the largest, n -th, outer ring and the second largest concentric ring is equal to r . The distance between the second largest and the third largest concentric rings is set equal to $2r$, etc. In this case the sum of all the inter-ring distances preceding the outer, n -th, concentric ring can also be found using the formula for the n -th term of the triangular number sequence due to the commutative property of addition. Therefore, modified matrix A of truncation term coefficients l^k from (2) for linearly decreasing inter-ring distances CRE is equal to:

$$A'' = \begin{pmatrix} n^4 & (2n-1)^4 & \dots & \left(\frac{n(n+1)}{2}\right)^4 \\ n^6 & (2n-1)^6 & \dots & \left(\frac{n(n+1)}{2}\right)^6 \\ \vdots & \vdots & \ddots & \vdots \\ n^{2n} & (2n-1)^{2n} & \dots & \left(\frac{n(n+1)}{2}\right)^{2n} \end{pmatrix} \quad (4)$$

C. FEM Modeling

To directly compare the discrete Laplacian estimates including the previously proposed constant inter-ring distances TCRE ($n = 2$) and QCRE ($n = 3$) configurations to their counterparts with variable inter-ring distances a FEM model from [9], [10], [17] was used with an evenly spaced square mesh size of 600×600 located in the first quadrant of the X - Y plane above a unit charge dipole projected to the center of the mesh and oriented towards the positive direction of the Z . Namely, comparisons to the linearly increasing and linearly decreasing variable inter-ring distances TCRE and QCRE configurations respectively were drawn. Bipolar CRE configuration ($n = 1$) was also included in the FEM model. To ensure direct comparability of results for different CRE configurations, all modeled CREs had the same dimensions despite having different numbers of rings. The largest, outer ring radius for all the CRE configurations was selected to be equal to $6r$ since 6 is the least common multiple of 2 and 3. Relative locations of concentric rings, with respect to the central disc, for all the TCRES and QCREs modeled are presented in Fig. 3. At each point of the mesh, the electric potential ϕ generated by a unity dipole was calculated with the formula for electric potential due to a dipole in a homogeneous medium of conductivity σ [18]:

$$\phi = \frac{1}{4\pi\sigma} \frac{(\vec{r}_p - \vec{r}) \cdot \vec{p}}{|\vec{r}_p - \vec{r}|^3} \quad (5)$$

where $\vec{r} = (x, y, z)$ and $\vec{p} = (p_x, p_y, p_z)$ represent the location and the moment of the dipole and $\vec{r}_p = (x_p, y_p, z_p)$ represents the observation point.

The analytical Laplacian was then calculated at each point of the mesh, by taking the second derivative of the electric potential ϕ [18]:

$$L = \Delta\phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \quad (6)$$

According to [18], this results in:

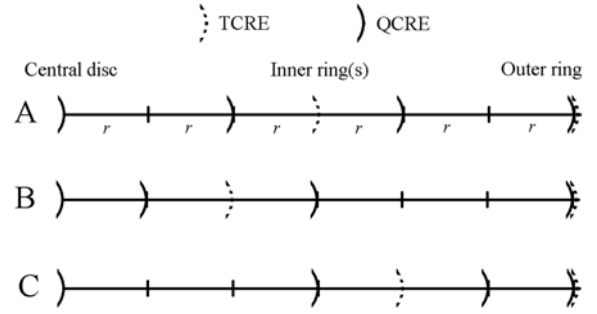


Figure 3. Relative locations of rings for constant (panel A), increasing (panel B) and decreasing inter-ring distances (panel C) TCRES and QCREs.

$$L = \frac{3}{4\pi\sigma} \left[5(z_p - z)^2 \frac{(\vec{r}_p - \vec{r}) \cdot \vec{p}}{|\vec{r}_p - \vec{r}|^7} - \frac{(\vec{r}_p - \vec{r}) \cdot \vec{p} + 2(z_p - z)p_z}{|\vec{r}_p - \vec{r}|^5} \right] \quad (7)$$

Laplacian estimates for seven CRE configurations were computed at each point of the mesh where appropriate boundary conditions could be applied. Modeling was repeated for different integer multiples of r ranging from 1 to 10. Since the model was tied to the physical dimensions (in cm) through the target physical size of the CRE, the smallest CRE diameter was equal to 0.5 cm (multiple of r equal to 1) and the largest was equal to 5 cm (multiple of r equal to 10). The dipole depth was equal to 5 cm.

Derivation of Laplacian estimate coefficients for variable inter-ring distances CRE configurations was performed using the approach proposed in this paper by finding the null space of matrices A' from (3) and A'' from (4) for $n = 2$ and $n = 3$. For TCRES the coefficients were $(81, -1)$ and $(81, -16)$ for increasing and decreasing inter-ring distances respectively. For QCREs the coefficients were $(4374, -70, 1)$ and $(6875, -2187, 625)$ for increasing and decreasing inter-ring distances respectively. Coefficients for constant inter-ring distances CRE configurations were adopted from [17]: $(16, -1)$ for TCRE and $(270, -27, 2)$ for QCRE. These seven estimates including three for TCRE (with constant, increasing, and decreasing inter-ring distances respectively), three for QCRE, and one for bipolar CRE configuration were then compared with the calculated analytical Laplacian for each point of the mesh where corresponding Laplacian estimates were computed using Relative Error and Maximum Error measures [9], [10], [17]:

$$\text{Relative Error}^i = \sqrt{\frac{\sum (\Delta v - \Delta^i v)^2}{\sum (\Delta v)^2}} \quad (8)$$

$$\text{Maximum Error}^i = \max |\Delta v - \Delta^i v| \quad (9)$$

where i represents the seven Laplacian estimation methods used to approximate the Laplacian potential $\Delta^i v$ and Δv represents the analytical Laplacian potential.

III. RESULTS

The FEM modeling results of two error measures computed for seven Laplacian estimation methods corresponding to seven CRE configurations using equations (8) and (9) respectively are presented on a semi-log scale in Fig. 4 for CRE diameters ranging from 0.5 cm to 5 cm.

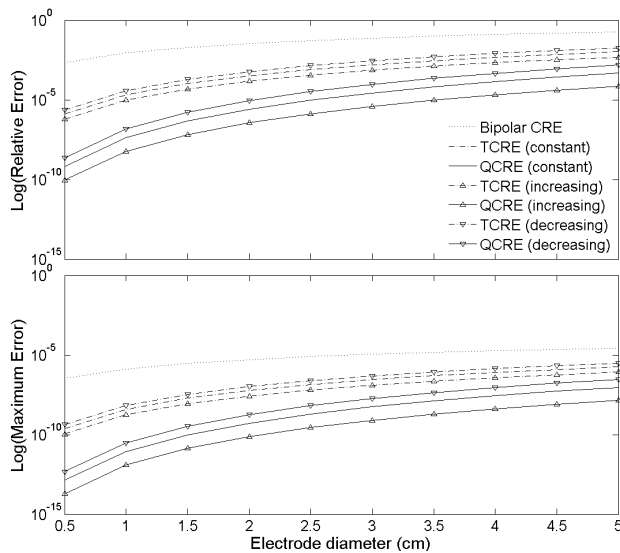


Figure 4. Relative (top panel) and Maximum (bottom) Errors of seven Laplacian estimates corresponding to bipolar CRE, TCREs, and QCREs.

Laplacian estimation errors in Fig. 4 suggest that the increasing inter-ring distances TCRE and QCRE configurations hold potential for an improvement over their constant inter-ring distances counterparts while the decreasing inter-ring distances TCRE and QCRE configurations do not. Moreover, improvement appears to become more significant with the increase of the number of rings (i.e. there is more improvement for QCREs than for TCREs). This stems from comparison of averages (mean \pm standard deviation) for 10 different sizes of each CRE configuration) of errors for constant inter-ring distances and increasing inter-ring distances CREs. For TCREs Relative and Maximum Errors are 2.23 ± 0.02 and 2.22 ± 0.03 times higher on average for constant inter-ring distances CREs respectively while for QCREs Relative and Maximum Errors are 6.95 ± 0.14 and 6.91 ± 0.16 times higher on average for constant inter-ring distances CREs respectively (Fig. 4).

IV. DISCUSSION

FEM modeling results obtained in this paper are consistent with the previous FEM results obtained for bipolar CREs and TCREs only [9,10] as well as for multipolar CREs [17] in terms of accuracy of Laplacian estimation increasing (Relative and Maximum Errors decrease) with an increase in the number of rings n and decreasing (Relative and Maximum Errors increase) with an increase in the diameter of the CRE. More importantly, obtained FEM modeling results suggest that increasing inter-ring distances CRE configurations may decrease Relative and Maximum Errors resulting in more accurate Laplacian estimates compared to respective constant inter-ring distances CRE configurations. More than two-fold and six-fold decreases in estimation error are expected for TCREs and QCREs respectively.

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