FINITE ELEMENT METHOD MODELING TO ASSESS AND COMPARE LAPLACIAN ESTIMATES VIA NOVEL MULTIPOLAR CONCENTRIC RING ELECTRODES

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Summary

This study uses finite element method model analysis to confirm the analytic result of the accuracy of Laplacian estimation using novel multipolar concentric ring electrodes increasing with the increase of the number of rings. Obtained results suggest significance of this gain in accuracy for practical applications.

Introduction

Conventional electroencephalography (EEG) with disc electrodes has major drawbacks including poor spatial resolution, selectivity and low signal-to-noise ratio that are critically limiting its use. Concentric ring electrodes (CREs), consisting of several elements including the central disc and a number of concentric rings, are a promising alternative that allows to improve all of the aforementioned aspects significantly. Because of such unique capabilities including superiority to conventional disc electrode, in particular, in accuracy of Laplacian estimation, the tripolar concentric ring electrodes (TCREs; the highest number of CRE poles currently used) have found numerous applications in a wide range of areas including brain-computer interface, seizure onset detection, seizure attenuation using transcranial focal stimulation applied via TCREs, etc [1]. Taking the next step toward further improving the Laplacian estimation with novel multipolar concentric ring electrodes we proposed a general approach to estimation of the Laplacian for an $(n+1)$-polar electrode with n rings using the $(4n+1)$-point method for $n \geq 2$ that allows cancellation of all the truncation terms up to the order of $2n$ [1]. This study confirms the analytic result of the accuracy of Laplacian estimate increasing with the increase of n and assesses the significance of this gain in accuracy for practical applications using finite element method (FEM) model analysis.

Methods

To compare the discrete Laplacian estimates including the bipolar (number of rings $n = 1$), tripolar ($n = 2$), quadripolar ($n = 3$), quintopolar ($n = 4$), sextopolar ($n = 5$), and septapolar ($n = 6$) multipolar CRE configurations directly a FEM computer model was developed with a 1700×1700 evenly spaced mesh located in the first quadrant of the X-Y plane above a unit charge dipole projected to the center of the mesh and oriented towards the positive direction of the Z axis. The dipole was moved along the Z axis to evaluate the effect of depth on accuracy of Laplacian estimates.

At each point of the mesh, the electric potential ϕ generated by a unity dipole was calculated with the formula for electric potential due to a dipole in a homogeneous medium of conductivity σ:

$$\phi = \frac{1}{4\pi\sigma} \frac{(\bar{r}_p - \bar{r}) \cdot \bar{p}}{|\bar{r}_p - \bar{r}|^3} \tag{1}$$

where $\bar{r} = (x, y, z)$ and $\bar{p} = (p_x, p_y, p_z)$ represent the location and the moment of the dipole and $\bar{r}_p = (x_p, y_p, z_p)$ represents the observation point. The analytical Laplacian was then calculated at each point of the mesh, by taking the second derivative of the electric potential ϕ:

$$L = \Delta\phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \tag{2}$$

Laplacian estimates for six multipolar CRE configurations ranging from bipolar to septapolar were computed at each point of the mesh where appropriate boundary conditions could be applied. The process was repeated for different interpoint distances using integer multiples of r ranging from 1 to 10 to evaluate the effect of the electrode size on accuracy of Laplacian estimates. The model was tied to the physical dimensions (in cm) through the target physical size of the multipolar CREs.

Derivation of Laplacian estimate coefficients for multipolar CRE configurations was performed using the approach proposed in [1]. These six estimates were then compared with the calculated analytical Laplacian using Relative Error measure:

$$\mathrm{RelErr}^i = \sqrt{\frac{\sum (\Delta v - \Delta^i v)^2}{\sum (\Delta v)^2}} \tag{3}$$

where i represents the Laplacian estimation method used to approximate the Laplacian potential $\Delta^i v$ and Δv represents the analytical Laplacian potential.

Full factorial design of analysis of variance (ANOVA) was used with three numerical factors. The first factor (A) was the dipole depth presented at five levels uniformly distributed in the range from 1 cm to 5 cm. The second factor (B) was the CRE diameter presented at ten levels uniformly distributed in the range from 0.5 cm to 5 cm. The third factor (C) was the number of rings in the multipolar CRE configuration presented at six levels ranging from one (bipolar CRE) to six (septapolar CRE). The response variable was the Relative Error computed for each of the $5 \times 10 \times 6 = 300$ combinations of levels of three factors.

Results

Relative Errors computed using (3) for different multipolar CRE configurations are presented on a semi-log scale in Figure 1. The effect of factors A, B, and C on Relative Error was assessed and the ANOVA results suggest that all three factors have statistically significant effects in the model (d.f. $= 3$, $F = 146.72$, $p < 0.0001$).

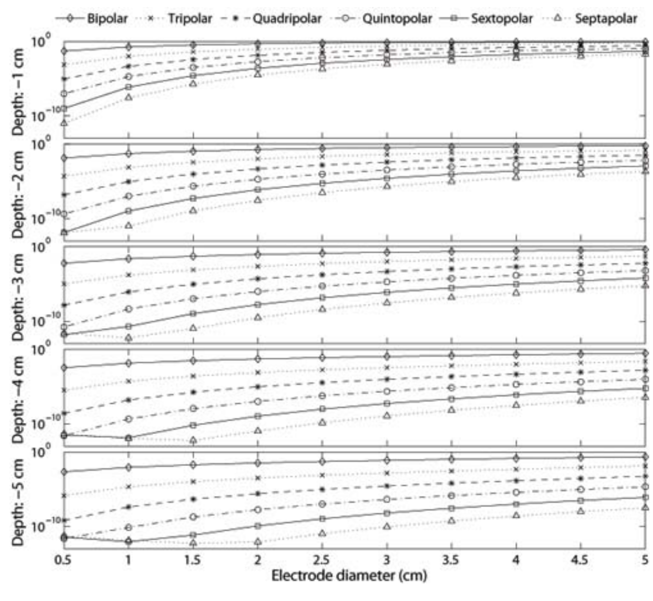

Figure 1: Error of six Laplacian estimates corresponding to different CRE configurations for multipolar CRE diameters ranging from 0.5 cm to 5 cm and dipole depth ranging from 1 cm to 5 cm.

Discussion

While it was important to confirm that the accuracy of Laplacian estimation increases (Relative Error decreases) with an increase of the dipole depth (factor A) and decreases (Relative Error increases) with an increase of the CRE diameter (factor B), the most important result is that, for the case of the factor C, the higher order multipolar Laplacian estimates are significantly better than the lower order ones at approximating the analytic Laplacian.

However, even though the statistical analysis showed that, in general, Relative Error was decreasing significantly with the increase in the number of CRE rings, the modeling results also suggest that for dipoles at higher depths and for smaller sizes of CREs the difference between the higher order multipolar CRE configurations becomes negligible. This can be seen from Fig. 1 at dipole depth of 3 cm and higher and the CRE diameter of 1 cm and lower. Existence of an upper bound on the maximum practical number of rings suggests the possibility of optimizing the design of a multipolar CRE for specific applications taking into account the requirements on the CRE size and expected maximum depth of the sources to record from.

The plan for future work is to use realistic head models to confirm the FEM modeling results obtained in this study. Unlike the currently used FEM model this approach would allow modeling the head as an irregular mesh of finite volumetric elements of various shapes and sizes [2]. Generating a realistic mesh of elements with similar tissue characteristics on an irregular grid is feasible, in part, thanks to increasing use of network science in neuroimaging [3].

References

[1] O. Makeyev, Q. Ding, S. Kay, and W. G. Besio. Toward improving the laplacian estimation with novel multipolar concentric ring electrodes. In *Proceedings of 35th Annual International Conference of the IEEE EMBS*, pages 1486–1489, Osaka, Japan, 2013.

[2] M. Rosenfeld, R. Tanami, and S. Abboud. Numerical solution of the potential due to dipole sources in volume conductors with arbitrary geometry and conductivity. *IEEE Transactions on Biomedical Engineering*, 43(7):679–689, 1996.

[3] Q. K. Telesford, S. L. Simpson, J. H. Burdette, S. Hayasaka, and P. J. Laurienti. The brain as a complex system: using network science as a tool for understanding the brain. *Brain connectivity*, 1(4):295–308, 2011.