Deriving Analytic Laplacian Estimates for Multipolar Concentric Ring Electrodes Using Inverse Vandermonde Matrix

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Concentric ring electrodes (CRE) have shown great promise in non-invasive electrophysiology.Tripolar CREsare successfully used in a wide range of applications including brain-computer interface and seizure onset detection among others, demonstrating their superiority to conventional disc electrodes, in particular, in accuracy of Laplacian estimation. Recently, we proposed a general approach to estimation of the Laplacian for an $(n + 1)$-polar electrode with n rings using the $(4n + 1)$-point method for $n \geq 2$ that allows cancellation of all the truncation terms up to the order of $2n$ which has been shown to be the highest order achievable for a CRE with n rings. This increase in accuracy of Laplacian estimation due to decrease in truncation error associated with an increase in $n$for novel multipolar CREs has been confirmed using finite element method modeling. The drawback of the proposed general approach was that for any $n \geq 2$ the Laplacian estimates in the form of the null space vectors could be calculated numericallyusing, for example, Gaussian elimination, but no analytic formula for Laplacian estimate as a function of n was derived. The goal of this study was to derive the explicit formula for multipolar Laplacian. This derivation was accomplished based on the inversion of a square Vandermonde matrix completing the proposed general approach for estimation of multipolar Laplacian and making it more efficient in terms of computation. This work is a part of our continued effort to improve the electrode design for non-invasive electrophysiology via multipolar CREs.